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EXCLUSION, DISCOVERY AND IDENTIFICATION OF DARK MATTER WITH DIRECTIONAL DETECTION

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Abstract. Directional detection is a promising search strategy to discover galactic Dark Matter. We present a Bayesian analysis framework dedicated to data from upcoming directional detectors. The interest of directional detection as a powerful tool to set exclusion limits, to authentify a Dark Matter detection or to constrain the Dark Matter properties, both from particle physics and galactic halo physics, will be demonstrated.

1 Introduction

Taking advantage on the astrophysical framework, directional detection of Dark Matter is an interesting strategy to distinguish WIMP events from background ones. Indeed, like most spiral galaxies, the Milky Way is supposed to be immersed in a halo of WIMPs which outweighs the luminous component by at least one order of magnitude. As the Solar System rotates around the galactic center through this Dark Matter halo, WIMPs should mainly come from the direction to which points the Sun velocity vector and which happens to be roughly in the direction of the Cygnus constellation ($\ell_\odot = 90°, b_\odot = 0°$). Then, a directional WIMP flux entering in any terrestrial detectors (see fig.1 left) should infer a directional WIMP induced recoil distribution pointing toward the Cygnus Constellation (see fig.1 middle). It corresponds to the expected WIMP signal probed by directional detectors which presents a strong anisotropy (Spergel 1988) while the background angular distribution should be isotropic. Hence, we argue that a clear and unambiguous identification of a Dark matter detection could be done by showing the correlation of the measured signal with the direction of the solar motion.

Several project of directional detectors are being developed (Daw *et al.* 2010, Miuchi *et al.* 2010, Santos *et al.* 2011, Ahlen *et al.* 2011, Vahsen *et al.* 2011, Naka *et al.* 2011).

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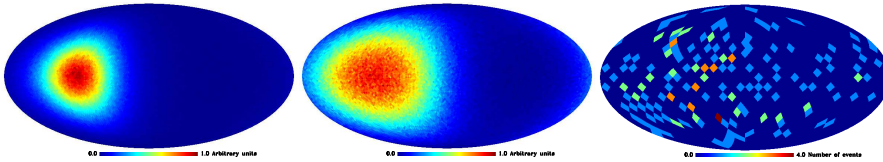
Fig. 1. From left to right : WIMP flux in the case of an isothermal spherical halo, WIMP-induced recoil distribution and a typical simulated measurement : 100 WIMP-induced recoils and 100 background events with a low angular resolution. Recoil maps are produced for a Fluorine target, a 100 GeV.c^{-2} WIMP and considering recoil energies in the range 5 keV $\leq E_R \leq$ 50 keV. Figures are taken from Billard *et al.* 2010a.

We present a complete Bayesian analysis framework dedicated to directional data. The first step when analysing directional data should be to look for a signal pointing toward the Cygnus Constellation with a sufficiently high significance. If no evidence in favor of a Galactic origin of the signal is deduced from the previous analysis, then an exclusion limit should be derived. In this paper, we consider three different scenarios which are, how to set robust and competitive exclusion limits, how to authentify a Dark Matter detection and to estimate the significance of the latter. Eventually, it is also possible to go further with directional detection by identifying the properties of the WIMP particle in the case of a high significance detection. Of course, those three scenarios depend on the value of the unknown WIMP-nucleon cross section and on the detector sensitivity. In the following, unless otherwise stated, we consider the following detector characteristics: a 10 kg of CF$_4$ detector with a recoil energy range of 5 keV $\leq E_R \leq$ 50 keV and a data acquisition time of 3 years.

2   Directional framework

Directional detection depends crucially on the WIMP velocity distribution. The isothermal sphere halo model is often considered but it is worth going beyond this standard paradigm in the case of model-independent analysis. Indeed, recent results from N-body simulations are in favor of triaxial Dark Matter haloes with anisotropic velocity distributions (Ling *et al.* 2010). Moreover, recent observations of Sagittarius stellar tidal stream have shown evidence for a triaxial Milky Way Dark Matter halo (Law *et al.* 2009).

The multivariate Gaussian WIMP velocity distribution corresponds to the generalization of the standard isothermal sphere with a density profile $\rho(r) \propto 1/r^2$, leading to a smooth WIMP velocity distribution, a flat rotation curve and no substructure. The WIMP velocity distribution in the laboratory frame is given by,

$$f(\vec{v}) = \frac{1}{(8\pi^3 \det \boldsymbol{\sigma}_v^2)^{1/2}} \exp\left[-\frac{1}{2}(\vec{v} - \vec{v}_\odot)^T \boldsymbol{\sigma}_v^{-2}(\vec{v} - \vec{v}_\odot)\right] \qquad (2.1)$$

where $\boldsymbol{\sigma}_v = \mathrm{diag}[\sigma_x, \sigma_y, \sigma_z]$ is the velocity dispersion tensor assumed to be diagonal in the Galactic rest frame $(\hat{x}, \hat{y}, \hat{z})$ and $\vec{v}_\odot$ is the Sun motion with respect to the Galactic rest frame. When neglecting the Sun peculiar velocity and the Earth orbital velocity about the Sun, $\vec{v}_\odot$ corresponds to the detector velocity in the Galactic rest frame and is taken to be $v_\odot = 220$ km.s^{-1} along the $\hat{y}$ axis pointing toward the Cygnus constellation at $(\ell_\odot = 90°, b_\odot = 0°)$. The velocity anisotropy $\beta(r)$, is then defined as

$$\beta(r) = 1 - \frac{\sigma_y^2 + \sigma_z^2}{2\sigma_x^2} \qquad (2.2)$$

According to N-body simulations, the β parameter at the Solar radius spans the range $0 - 0.4$, corresponding to radial anistropy.

In the following, Unless otherwise stated, we consider the standard halo model to generate simulated data, *i.e.* an isotropic velocity distribution ($\beta = 0$) in which case the velocity dispersions are related to the local circular velocity $v_0 = 220$ km/s as $\sigma_x = \sigma_y = \sigma_z = v_0/\sqrt{2}$.

The directional recoil rate is given by (Gondolo 2002) :

$$\frac{\mathrm{d}^2 R}{\mathrm{d}E_R \mathrm{d}\Omega_R} = \frac{\rho_0 \sigma_0}{4\pi m_\chi m_r^2} F^2(E_R) \hat{f}(v_{\min}, \hat{q}), \qquad (2.3)$$

with m_χ the WIMP mass, m_r the WIMP-nucleus reduced mass, $\rho_0 = 0.3$ GeV/c^2/cm^3 the local Dark Matter density, σ_0 the WIMP-nucleus elastic scattering cross section, $F(E_R)$ the form factor (using the axial expression from (Lewin and Smith 1996)) and $v_{\min}$ the minimal WIMP velocity required to produce a nuclear recoil of energy E_R. Finally, $\hat{f}(v_{\min}, \hat{q})$ is the three-dimensional Radon transform of the WIMP velocity distribution $f(\vec{v})$. Using the Fourier slice theorem (Gondolo 2002), the Radon transform of the multivariate Gaussian is,

$$\hat{f}(v_{\min}, \hat{q}) = \frac{1}{(2\pi \hat{q}^T \boldsymbol{\sigma}_v^2 \hat{q})^{1/2}} \exp\left[-\frac{[v_{\min} - \hat{q}.\vec{v}_\odot]^2}{2\hat{q}^T \boldsymbol{\sigma}_v^2 \hat{q}} \right]. \qquad (2.4)$$

3 Case of a null detection

We present a Bayesian estimation of exclusion limits dedicated to directional data where only the angular part of the event distribution is considered Billard *et al.* 2010b. The fact that both signal and background angular spectra are well known allows to derive upper limits using the Bayes' theorem. Considering an extended likelihood function with flat priors for both the expected number of WIMP events (μ_s) and background events (μ_b), and taking the evidence as a normalization factor, it is reduced to

$$\mathscr{L}(\vec{\theta}) = \frac{(\mu_s + \mu_b)^N}{N!} e^{-(\mu_s + \mu_b)} \times \prod_{n=1}^{N} \left[\frac{\mu_s}{\mu_s + \mu_b} S(\vec{R}_n) + \frac{\mu_b}{\mu_s + \mu_b} B(\vec{R}_n) \right] \quad (3.1)$$

where $\vec{\theta} = \{\mu_s, \mu_b\}$, $\vec{R}_n$ refers to the characteristics of the events, direction and energy and N corresponds to the total number of observed events. Hence, the probability density function of the parameter of interest μ_s can be derived by marginalizing $\mathscr{L}(\vec{\theta})$ over the parameter μ_b. The excluded number of WIMP events μ_{exc}, corresponding to an excluded cross-section, at 90% CL is obtained by solving:

$$\int_0^{\mu_{\text{exc}}} \mathscr{L}(\mu_s) \, d\mu_s = 0.9, \tag{3.2}$$

For each detector configuration and input, we have used 10 000 toy Monte Carlo experiments in order to evaluate the frequency distributions of the excluded cross-section. Then, from each distribution, we can derive the median value of the excluded cross-section σ_{med}. More details may be found in (Billard *et al.* 2010b). In the following, we have considered the effect of three experimental issues on exclusion limits using directional detection: background contamination, angular resolution and the sense recognition capability. Indeed, even though several progresses have been done, these experimental issues remain challenging for directional detection of Dark Matter. On the left panel of figure 2, we present the exclusion limits in the (m_χ, σ_n) plane for four different cases: background free (black solid line), ideal detector with a background contamination of 10 events/kg/year (black dotted line), the effect of a $45°$ degree angular resolution (red solid line) and the effect of no sense recognition (blue solid line). It should be noticed that in the case of angular resolution and sense recognition, a background rate of 10 events/kg/year is also considered.

From the left panel of figure 2, it can be seen that the main experimental issues when setting exclusion limits with directional detection is obviously the background contamination. However, with a background contamination of 300 expected events, if one do not use directional information (Poisson limit), the exclusion limit should be two order of magnitude above the one corresponding to the background free configuration. Then, we can deduce from the left panel of figure 2 that directional information allows us to improve exclusion limits by about one order of magnitude, highlighting the interest of this kind of direct detection.

Without sense recognition, a recoil coming from $(\cos\gamma, \phi)$ cannot be distinguished from a recoil coming from $(-\cos\gamma, \phi + \pi)$ leading to an expected angular distribution, from WIMP events, less anisotropic and then closer to the one from background events. One can see from the left panel of figure 2 that the effect of having or not the sense recognition capability in the case of high background contamination will only affect the exclusion limit by a factor of ~ 4. Taken at face value, this result suggests that sense recognition may not be so important for directional detection when setting exclusion limits.

Having a finite angular resolution means that a recoil initially coming from the direction $\hat{r}(\theta, \phi)$ is reconstructed as a recoil $\hat{r}'(\theta', \phi')$ with a gaussian dispersion of width σ_Θ. Then, the effect of angular resolution is that the angular distribution is smoother and hence degrades the discrimination between the expected WIMP and background events. The effect of an angular resolution of $\sigma_\Theta = 45°$ at high background contamination can be seen by comparing the red solid line and the
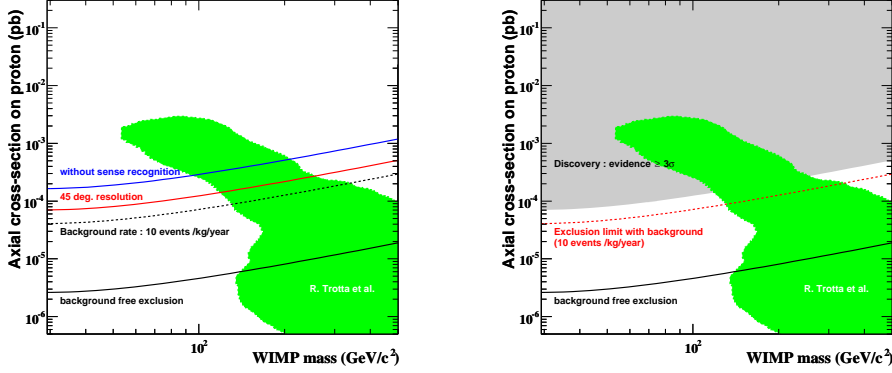
Fig. 2. Spin dependent cross section on proton (pb) as a function of the WIMP mass (GeV/c^2), in the pure-proton approximation. The green shaded area represent the favored region obtain in the constrained minimal supersymmetry. Left: projected exclusion limits of a forthcoming directional detector of 30 kg.year are presented in four cases : background-free (black solid line), with a background rate of 10 events/kg/year with sense recognition and perfect angular resolution (black dotted line), with the same background rate considering $\sigma_\Theta = 45°$ (red solid line) and considering no sense recognition (blue solid line). Right: the shaded area presents the 3σ discovery region.

black dotted one from the left panel of figure 2. One can see that an angular resolution of 45° only degrade the exclusion limit of a factor ~ 2 for a background contamination of 10 events/kg/year. Hence, as far as exclusion limits are concerned, the effect of angular resolution is relatively small.

To conclude this study, one can see from the left panel of figure 2 that directional detection should be able to reach a large fraction of some supersymmetric model, highlighting the need for this kind of direct detection of Dark Matter.

4   Case of positive detection

An observed recoil map such as the one on the right of figure 1 could be obtained with a 10 kg CF$_4$ detector with a WIMP-nucleon cross-section of $\sigma_n = 1.5 \times 10^{-3}$ pb and with a background rate of ~ 0.07 kg^{-1}.day^{-1} in ~ 5 months exposition time. At first sight, it seems difficult to conclude from this simulated recoil map that it does contain a fraction of WIMP events pointing toward the direction of the Solar motion. A likelihood analysis is developed (Billard *et al.* 2010a) in order to retrieve from a recoil map : the main direction of the incoming events in Galactic coordinates (ℓ, b) and the number of WIMP events contained in the map. The likelihood value is estimated using a binned map of the overall sky with

Poissonian statistics, as follows :

$$\mathscr{L}(m_\chi, \lambda, \ell, b) = \prod_{i=1}^{N_{\mathrm{pixels}}} P([(1-\lambda)B_i + \lambda S_i(m_\chi; \ell, b)]|M_i) \qquad (4.1)$$

where B is the background spatial distribution taken as isotropic, S is the WIMP-induced recoil distribution and M is the measurement. This is a four parameter likelihood analysis with m_χ, $\lambda = \mu_s/(\mu_s + \mu_b)$ the WIMP fraction (related to the background rejection power of the detector) and the coordinates (ℓ, b) referring to the maximum of the WIMP event angular distribution. Hence, $S(m_\chi; \ell, b)$ corresponds to a rotation of the $S(m_\chi)$ distribution by the angles ($\ell' = \ell - \ell_\odot$, $b' = b - b_\odot$). A scan of the four parameters with flat priors, allows to evaluate the likelihood between the measurement (fig. 1 right) and the theoretical distribution made of a superposition of an isotropic background and a pure WIMP signal (fig. 1 middle). By scanning on ℓ and b values, we ensure that there is no prior on the direction of the center of the WIMP-induced recoil distribution. As the observed map is considered as a superposition of the background and the WIMP signal distributions, no assumption on the origin of each event is needed. Moreover, the likelihood method allows to recover λ, the WIMP fraction contained in the data.

In order to explore the interest of directional detection combined with such likelihood analysis, we have done some systematical studies using 10^4 experiments for various number of WIMP events (N_{wimp}) and several values of WIMP fraction in the observed map (λ), ranging from 0.1 to 1. For a given cross-section, these two parameters are related respectively with the exposure and the rejection power of the offline analysis preceding the likelihood method.
Figure 3 presents on the left panel the directional signature, taken as the value of $\sigma_\gamma = \sqrt{\sigma_\ell \sigma_b}$, the radius of the 68% CL contour of the marginalised $\mathscr{L}(\ell, b)$ distribution, as a function of λ. It is related to the ability to recover the main signal direction and to sign its Galactic origin. It can be noticed that the directional signature is of the order of 10° to 20° on a wide range of WIMP fractions. Even for low number of WIMPs and for a low WIMP fraction (meaning a poor rejection power), the directional signature remains clear. From this, we conclude that a directional evidence in favor of Galactic Dark Matter may be obtained with upcoming experiments even at low exposure and with a non-negligible background contamination.
However, a convincing proof of the detection of WIMPs would require a directional signature with sufficient significance. We defined the significance of this identification strategy as λ/σ_λ, presented on figure 3 (right panel) as a function of λ. As expected, the significance is increasing both with the number of WIMP events and with the WIMP fraction, but we can notice that an evidence (3σ) or a discovery (5σ) of a Dark Matter signal would require either a larger number of WIMPs or a lower background contamination.
To conclude, on the right panel of figure 2 we have presented the area in the (m_χ, σ_n) plane for wich a directional detector, with 30 kg.year exposure, could
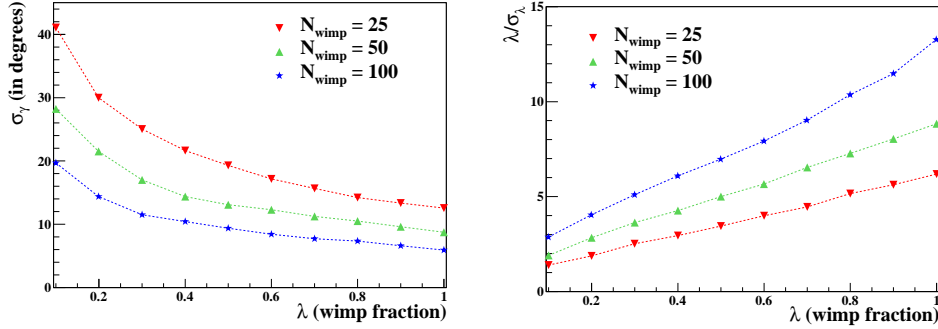
Fig. 3. Left panel presents the directional signature σ_γ (in degrees) as a function of the WIMP fraction. Right panel presents the significance (λ/σ_λ) as a function of the WIMP fraction $\lambda = \mu_s/(\mu_s + \mu_b)$. Results are produced for a ^{19}F target, a 100 GeV.c^{-2} WIMP and considering recoil energies in the range 5 keV $\leq E_R \leq$ 50 keV. Figures are taken from Billard *et al.* 2010a.

reach a 3σ significance detection in average. Then, a directional detector as the one considered here, should be able to reach a 3σ significance detection of Dark Matter down to a WIMP-nucleon cross section of about 10^{-4} pb.

5   Identification of Dark Matter

As we have seen previously, directional detection should provide powerful arguments in order to authentify a Dark Matter detection, or to set robust and competitive exclusion limits in the case of a null detection. However, it is possible to go further by exploiting all the information from a directional detector, *i.e.* the energy and the direction of each event. We will then consider the most optimistic scenario where the WIMP nucleon cross-section is sufficiently large to get a high significance Dark Matter detection. Then, we show for the first time the possibility to constrain the WIMP properties, both from particle physics (m_χ, σ_n) and galactic Dark Matter halo physics (velocity dispersions) (Billard *et al.* 2011). This leads to an identification of non-baryonic Dark Matter, which could be reached within few years by upcoming directional detectors (Ahlen *et al.* 2010). The model is characterized by 8 free parameters which are $\{m_\chi, \log_{10}(\sigma_n), l_\odot, b_\odot, \sigma_x, \sigma_y, \sigma_z, R_b\}$, where the direction $(l_\odot, b_\odot)$ refers to the main direction of the recorded events (see sec.4), σ_n is the WIMP-nucleon cross section directly related to σ_0 in the pure proton approximation and R_b is the background rate. We have considered flat prior for each parameter. In such case, the Bayes' theorem is simplified and the target distribution reduces to a 8 dimensional likelihood function $\mathscr{L}(\vec{\theta})$ dedicated to unbinned data as given by equation (3.1). The latter is sampled by using a MCMC analysis based on the Metropolis-Hastings algorithm, using chain subsampling according to the burn-in and correlation lengths to deal only with independent samples. More details may be found in (Billard *et al.* 2011).
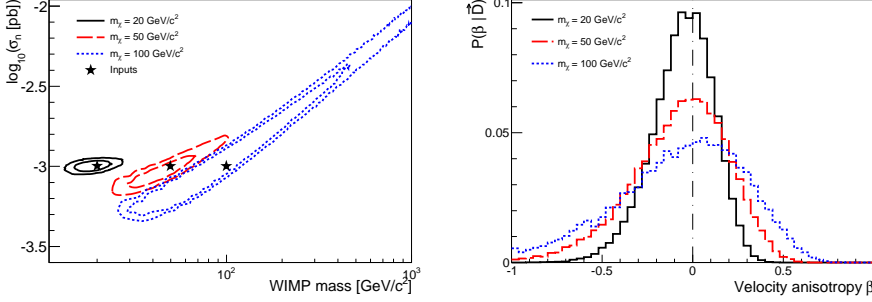
Fig. 4. Left panel : 68% and 95% contour level in the (m_χ, σ_n) plan, for the isotropic input model and for a WIMP mass equal to 20, 50 and 100 GeV/c^2. Right panel : posterior PDF distribution of the β parameter for the same models. Figures are taken from Billard *et al.* 2011.
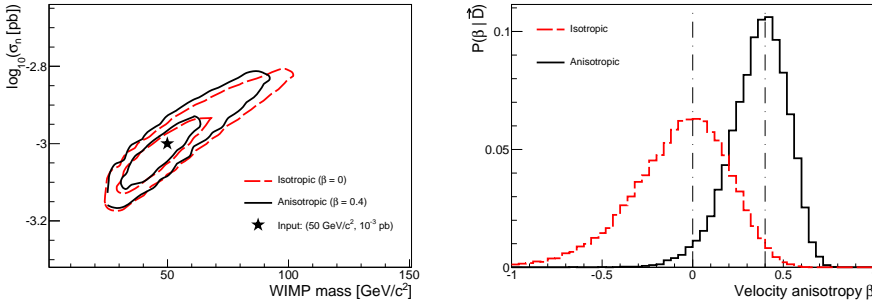
Fig. 5. Left panel : 68% and 95% contour level in the (m_χ, σ_n) plan, for a 50 GeV/c^2 WIMP and for two input models : isotropic ($\beta = 0$) and triaxial ($\beta = 0.4$). Right panel : posterior PDF distribution of the β parameter for the same models. Figures are taken from Billard *et al.* 2011.

In the following, we briefly discuss the effect of some of the input parameters on the different constraints in order to estimate the performance of this analysis tool. We first focus on the impact of the input WIMP mass. To do so, we have simulated three different sets of directional data corresponding to an input WIMP mass of $m_\chi = 20, 50, 100$ GeV/c^2 with a constant WIMP-nucleon cross-section $\sigma_n = 10^{-3}$ pb and the standard isotropic halo model. The results from the three MCMC runs are illustrated on figure 4. We present for the three WIMP masses, on the left panel, the 68% and 95% CL contours in the $(m_\chi, \log_{10}(\sigma_n))$ plan and on the right panel, the posterior PDF $P(\beta|\vec{D})$ of the anisotropy velocity parameter β.

It can be deduced from figure 4 that the constraints strongly depend on the input WIMP mass, but in each case, the constraints are consistent with the input val-

ues. Then, this analysis has been shown to be working for any input WIMP mass although the constraints are stronger for light WIMPs. This is due to the fact that the signal characteristics, *i.e* the slope of the energy distribution and the width of the angular distribution, evolve slowly with the WIMP mass once $m_\chi \geq 100$ GeV/c^2, as shown in (Billard *et al.* 2010a).

In the following, we investigate the effect of an extremely triaxial halo model with $\beta = 0.4$ (*i.e.* $\sigma_x = 200$ km/s; $\sigma_z = 169$ km/s; $\sigma_y = 140$ km/s) on the estimation of the Dark Matter parameters $(m_\chi, \sigma_n, \beta)$. The results from the MCMC run on a simulated dataset corresponding to a WIMP mass of 50 GeV/c^2 with the anisotropic halo model are presented on figure 5. For convenience and comparison, the results from a benchmark input model (isothermal sphere with a 50 GeV/c^2 WIMP) are recalled.

From the left panel of figure 5, we can conclude that the two halo models give similar constraints which are both consistent with the input values. In fact, and as foreseen, the fact that the velocity dispersions are set as free parameters in the MCMC analysis allows to avoid induced bias due to wrong model assumption. From the right panel of figure 5 we can deduce that the β parameter is well constrained: $\beta = 0.38^{+0.2}_{-0.1}$ and strongly in favor of an anisotropic Dark Matter halo. As one can see, directional detection should provide strong constraints on Dark Matter both from particle physics (mass and cross section) and also from astrophysics. Indeed, the fact that it is possible to constrain the local WIMP velocity distribution gives the opportunity to estimate the Dark Matter halo profile and start a Dark Matter astronomy.

6   Conclusion

As a conclusion, it can be highlighted that Directional detection should provide unambiguous arguments in favor of a positive or a null Dark Matter detection. Indeed, the angular distribution of the expected WIMP events is very unlickely to be mimicked by known backgrounds. That way, we have shown that Bayesian analysis of directional data are suitable to either set exclusion limits in the case of a null detection or to clearly authentify a positive detection. Moreover, we have seen that in the case of a high significance detection of Dark Matter, directional detection should be able to probe the structure of the local WIMP velocity distribution and to constrain the WIMP mass and cross section at the same time.

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